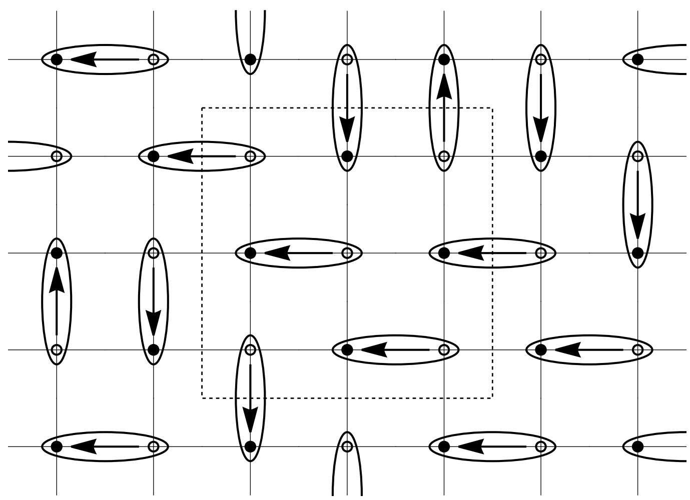
Homework for "Topological spin liquids I"

Gábor B. Halász[1,2]

[1]*Materials Science and Technology Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA*
[2]*Quantum Science Center, Oak Ridge, TN 37831, USA*

Here, we will consider a resonating valence-bond (RVB) state on the square lattice, and understand how the arguments in the lecture generalize to this different lattice geometry. The key difference of the square lattice with respect to the triangular lattice is its bipartite structure; the sites of the lattice can be divided into two sublattices A and B, denoted by white and black colors in the figure below, such that all neighbors of A sites are B sites and vice versa. Therefore, each spin-singlet dimer connects an A site with a B site and is endowed with a natural orientation, which we can represent with an arrow pointing from the A site to the B site. For a given boundary, we can then define the "charge" of each valence-bond (VB) state as $n = n_{\text{out}} - n_{\text{in}}$, where n_{in} and n_{out} are the numbers of spin-singlet dimers crossing the boundary with arrows pointing in and out, respectively. For example, in the figure below, $n_{\text{in}} = 2$ and $n_{\text{out}} = 3$, hence $n = n_{\text{out}} - n_{\text{in}} = 1$.



Exercise 1. Show that, for any given boundary, all possible VB states have charge $n = n_A - n_B$, where n_A and n_B are the numbers of A and B sites inside the boundary.

Exercise 2. Consider a transformation where the coefficient of each VB state within the RVB state is multiplied by $e^{i\alpha n}$ in terms of a real number $0 \leq \alpha < 2\pi$ and the charge n of the given VB state, as defined above. For which values of α does this transformation leave the RVB state invariant? Why is the RVB state on the square lattice called a $U(1)$ spin liquid?

It is important to note that pure $U(1)$ spin liquids are actually unstable in two spatial dimensions. However, they can be stable when coupled to gapless fermions with Dirac points or Fermi surfaces. Thus, two-dimensional "Dirac $U(1)$" and "Fermi $U(1)$" spin liquids are actively discussed in the literature.